Meteorology

- Simple meteorological models
  – Obtain concentration of species
    - Mass balance equation
      - Box model
      - Puff models
- Winds
  – Transport in the atmosphere
    - Forces affecting the wind
    - Flow patterns around high and low pressures
    - Atmospheric stability
    - General circulation → climate zones

Literature connected with today's lecture:
Jacob, chapter 3 - 4
Exercises:
3:1 – 3:5; 4:1 – 4:5

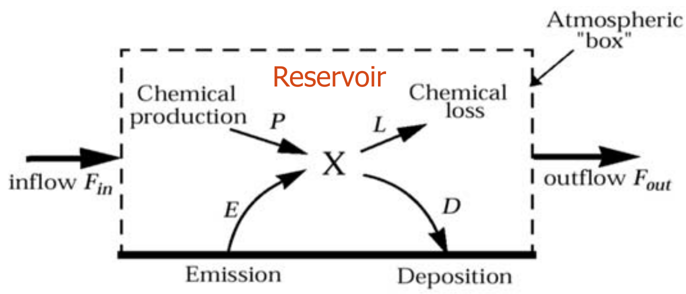
Simple Meteorological Models

Atmospheric concentration is controlled by:

- **Emission (E):** natural and anthropogenic sources
- **Deposition (D):** dry and wet deposition
- **Transformation (P,L):** chemical reactions and phase transitions
- **Transport (F):** winds

Box model (Eulerean)

- Imaginary box in the atmosphere
- Homogeneous concentration of X in the box is assumed in the model
  - Amount of species X
  - Sources: E, P, F_{in}
  - Sinks: D, L, F_{out}
  - Dimension: mass/time, with X amount as mass



Box Model – The Mass Balance Equation

$dm/dt = \Sigma\text{sources} - \Sigma\text{sinks} = F_{in} + E + P - F_{out} - D - L$

- Residence time in a box
  - Average time a species spends in the box
  - $\tau = m/(F_{out} + L + D)$ ([mass in the box]/[outflow])

- First order sinks
  - F_{out}, L and D often proportional to the amount in the box
  - $F_{out} + L + D = (k_{out} + k_l + k_d)m = km$
  - Loss rate constant: $k = (F_{out} + L + D)/m = 1/\tau$

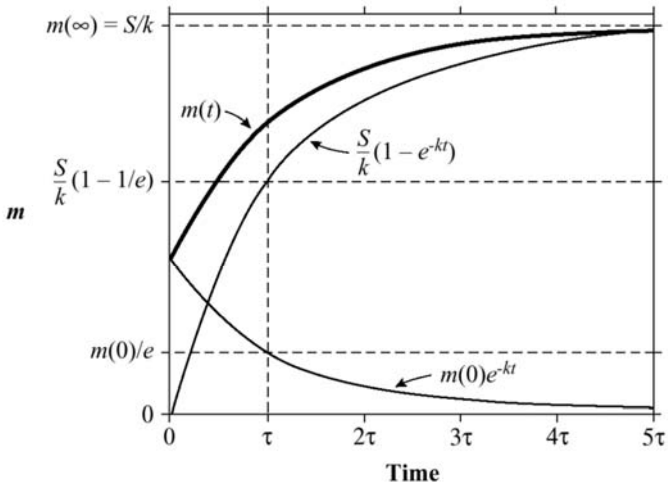
Box Model – Common Special Case

- Sources independent of m
- Sinks proportional to m

Mass balance equation:

dm/dt = Σsources - Σsinks =

= F_{in} + E + P - F_{out} - D – L



F_{in} + E + P = S (S indep. of m)

F_{out} + D + L = km (sinks prop. m)

dm/dt = S - km

Compute m(t)!

$\int dm/(S-km) = \int dt \Rightarrow$

$m(t) = m(0)e^{-kt} + S/k(1-e^{-kt})$

$t \to \infty \;\Rightarrow m \to S/k$

Exercise 3-3 in Jacob: The sink of CFC-12 (CF_2Cl_2) is exclusively photolysis (residence time 100 years). Year 1980 the concentration was 400 pptv and the rate of increase 4% per year. Calculate the 1980 CFC-12 emission!

Use the mass balance equation:

$$\frac{dm}{dt} = F_{in} + E + P - F_{ut} - L - D$$

Box: Entire atmosphere $\Rightarrow F_{in} = F_{out} = 0$
L + D = L = km (sink: photolysis only)
E + P = E (no chemical production)

$$\frac{dm}{dt} = E - km$$

dm/dt is given by a relative measure: k_s = 4% per year):

$$\frac{dm}{dt} = k_s \times m$$

Sink (photolysis): $k = 1/\tau = 0.01\ \text{year}^{-1}$

Emissions (from mass balance equation):

$$E = \frac{dm}{dt} + km = (k_s + k)m$$

Enter numbers: E = (0.04 year^{-1} + 0.01 year^{-1}) m

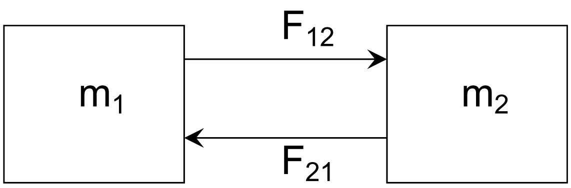
$$m = M_{CFC} n_{CFC} = \left[C_{CFC} = \frac{n_{CFC}}{n_a} \right] = M_{CFC} C_{CFC} n_a = M_{CFC} C_{CFC} \frac{m_a}{M_a} = M_{CFC} C_{CFC} \frac{4\pi R^2 P}{M_a g}$$

Result: E = 4.4×10^8 kg/year

Multiple Box Models

- A single box model sometimes oversimplifies a problem
- Multi-box models allow concentration to vary between boxes

Two-box Model



- Mass balance equation for box 1:

$$\frac{dm_1}{dt} = E_1 + P_1 - L_1 - D_1 - F_{12} + F_{21}$$

- First order process:
  - $F_{12} = k_{12}m_1$
  - $F_{21} = k_{21}m_2$
- Coupled differential equations:

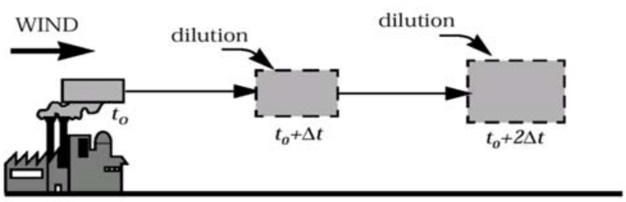
$$\frac{dm_1}{dt} = E_1 + P_1 - L_1 - D_1 - k_{12}m_1 + k_{21}m_2$$

$$\frac{dm_2}{dt} = E_2 + P_2 - L_2 - D_2 + k_{12}m_1 - k_{21}m_2$$

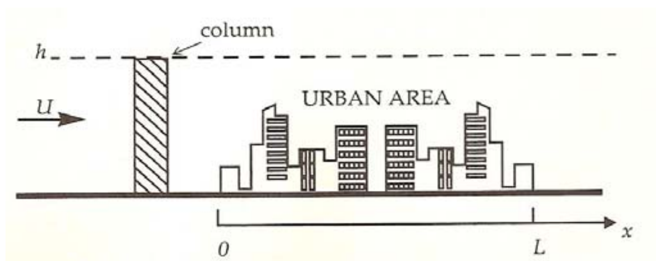
Lagrangean Model

- **Box model:** Air flows in and out of the box (Eulerean model)
- **Puff models:** Follows an air parcel in the atmosphere (Lagrangean model)

Smoke plume



- Mass bal. eqn. Lagrangean model:
  - $d[X]/dt = E + P - D - L$
  - Advantage: $F_{in} = F_{out} = 0$
  - Disadvantage: Limited range due to turbulence that diffuses the air parcel
- Common applications:
  - Smoke plumes
    - $d[X]/dt = E+P-D-L-k_{dil}([X]-[X]_b)$
    - k_{dil} = dilution constant
  - Column model
    - $d[X]/dt = E/h+P-D-L$
    - E = emission per area and time unit
    - h = height of the column (mixing height)



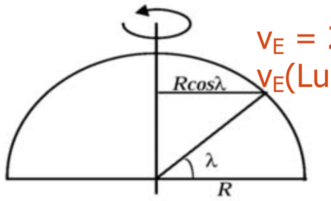
Transport in the Atmosphere

Forces that affect winds:

- Gradient force - Horizontal pressure gradient, strong winds (high-, low pressure)
- Gravitational force - induces vertical air motions related to density
- Coriolis force – Caused by the rotation of the earth
- Friction force – Acts on winds in contact with the ground

Coriolis Force

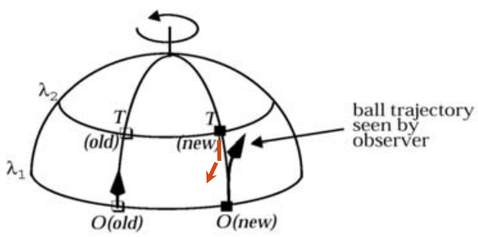
- Rotation of the earth
  - An object at the surface has velocity v_E
  - v_E depends on latitude



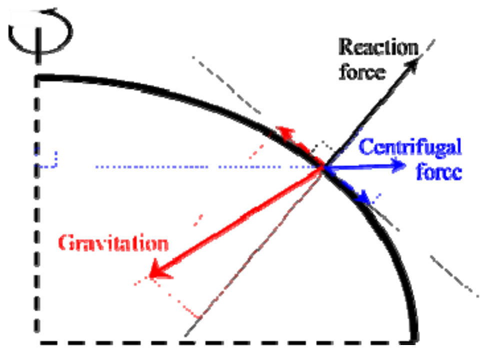
$v_E = 2\pi R\cos\lambda/t$

$v_E(\text{Lund}) \approx 200$ m/s

- Coriolis force along longitude



- Coriolis force along latitude



Rest: Horizontal comp. cancel out

$\Rightarrow$ The earth flattened at the poles

Motion west - east at northern hemisphere:
$\Rightarrow$ increased velocity $\Rightarrow$ increased centrifugal force $\Rightarrow$
Bends towards the equator (right)
East - west $\Rightarrow$ bends towards the north pole (right)
Southern hemisphere: bends to the left

Winds both along longitude and latitude bends
Northern hemisphere: Bends to the right   Southern hemisphere: To the left

Coriolis Force

- Coriolis acceleration

$$\gamma_c = 2\omega\nu\sin\lambda$$

- Resulting displacement

$$\Delta Y = \omega(\Delta X)^2\sin\lambda/\nu$$

$\omega = 2\pi/T$ (angular velocity)
ν = velocity relative the earth
ΔX = distance
λ = latitude

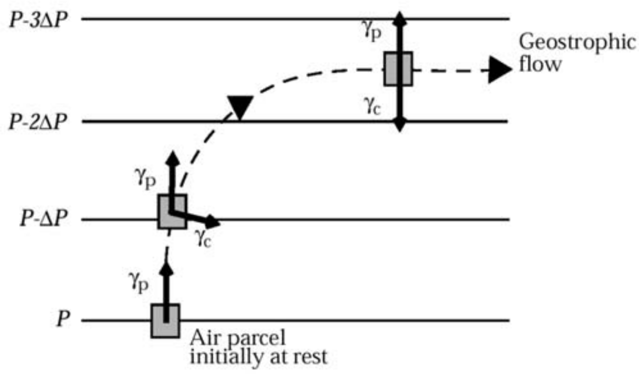
Example ($\lambda = 42°$):

1. Snow ball ν = 20 km/h ΔX = 10 m: ΔY = 1mm
2. Missile ν = 2000 km/h ΔX = 1000 km: ΔY = 100km

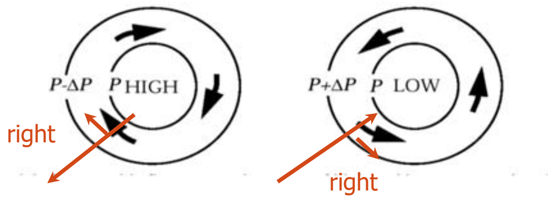
Geostrophic Wind

- Pressure gradient induces a wind

Motion $\Rightarrow$ Coriolis force
  - Balance between gradient and Coriolis forces $\Rightarrow$
- Geostrophic wind – Wind along isobar $\Rightarrow$
- No air transport to/from low/high pressure!

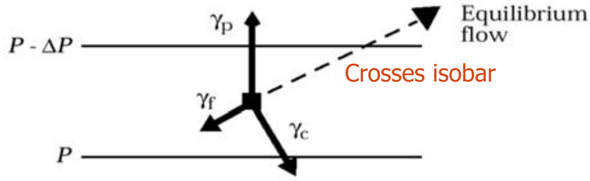


- Geostrophic wind around high and low pressures (northern hemisphere)



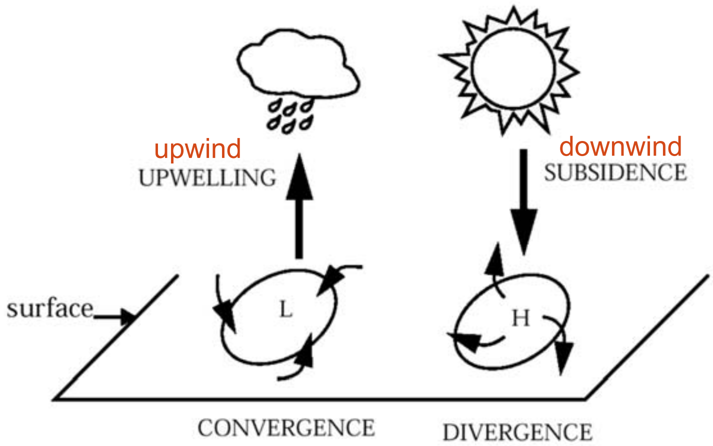
Friction Force

- Effect of friction against the ground



- High and low pressures
  - Wind to low pressures close to the ground
    - Convergence - Upwind in low pressures ⇒
    - Air expands and cool ⇒ Clouds
  - Wind from high pressure close to the ground
    - Divergence - Subsidence in high pressures ⇒
    - Air compressed - warmed ⇒ Clear weather

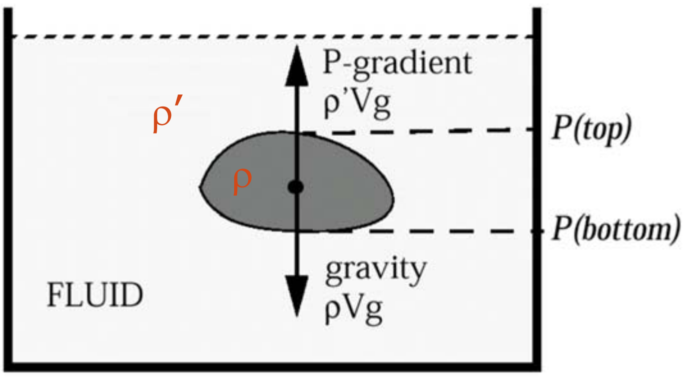
- Friction acts against the motion and reduces the speed
- ⇒ reduced Coriolis force
- ⇒ wind component crosses isobars close to the ground



Vertical Transport

- **A fluid at equilibrium:** The force from a pressure gradient acting on a volume element is balanced by the gravitational force
  - Gradient force: $F_g = \rho' Vg$
- Convection (buoyancy)
  - Caused by difference in density :
  - Gravitational force: $mg = -\rho Vg$
- Resulting force:

$$F = (\rho' - \rho)Vg$$



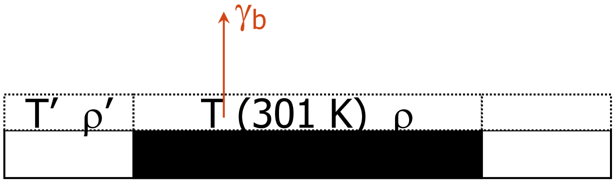
Buoyant acceleration:

$$\gamma_b = F/m = (\rho' - \rho)g/\rho$$

Exercise 4-1 (Jacob)

Assume that the air surface temperature over a black parking lot is 1 K higher than in the surrounding air (300 K).

Calculate the buoyant acceleration!



Bouyant acceleration: $\gamma_b = (\rho' - \rho) g / \rho$

From the ideal gas law: $\rho = PM_a / RT$

$\gamma_b = (T - T') g / T'$

$\gamma_b = 0.033$ m/s^2

Calculate the upward wind speed after 1 s?

$w = t\,\gamma_b = 3.3$ cm/s

Comparison:

Global circulation: $w \approx 0.1$ cm/s

Cumulus clouds: w several m/s

Does the velocity continue to increase? → **Atmospheric stability**

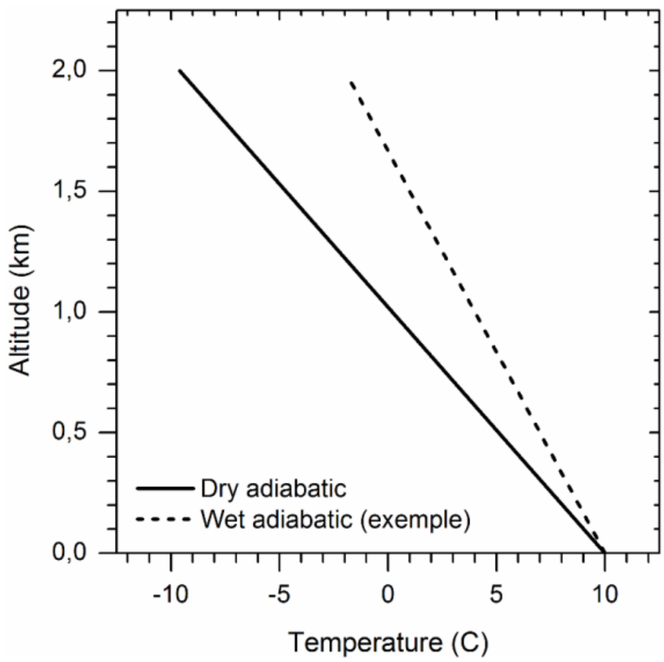
Adiabatic Lapse Rate

- Small heat exchange in vertical transport ($dQ \approx 0$) ⇒ Adiabatic process
  - Upwards ⇒ expansion ⇒ cooling
  - Downwards ⇒ compression ⇒ warming
- Dry adiabatic lapse rate

  Can be shown (see the text book):

  $\Gamma = -dT/dz = g/C_p = 9.8$ K/km
- Wet adiabatic lapse rate

  Cloud formation ⇒ heat of condensation released ⇒ weaker gradient; Γ_w 2-7 K/km

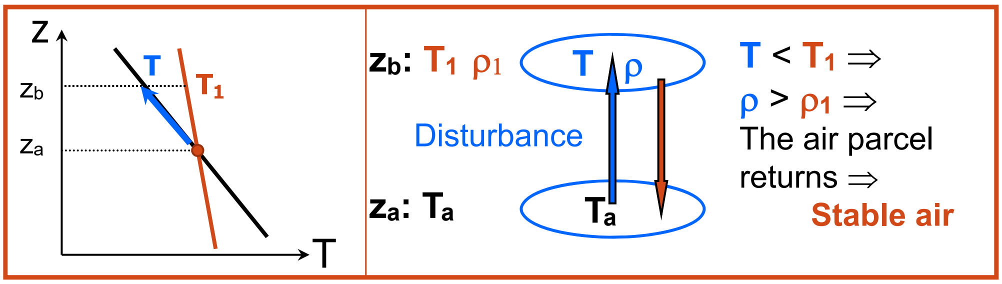


Air masses in vertical motion follow the adiabatic lapse rate (dry or wet)

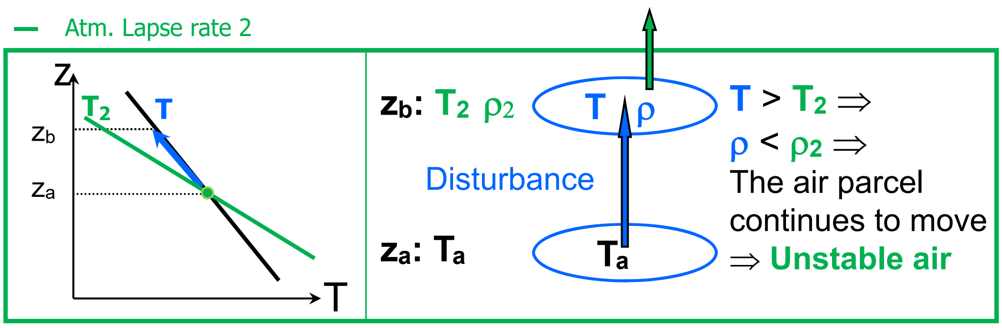
Atmospheric Stability

Sometimes strong vertical motions and turbulence – sometimes not – WHY?

Disturbance: An mass air at height z_a is lifted to $z_b \Rightarrow$ Expands and cools adiabatically to **T**



— Atm. Lapse rate 1
— Dry adiabatic lapse rate
— Atm. Lapse rate 2



At z_a (initially): $\mathbf{T_a}$

At z_b:

Disturbed air mass:

$\mathbf{T} = T_a - \Gamma(z_b - z_a)$

Surrounding atmosphere:

Case 1: $\mathbf{T_1}$

Case 2: $\mathbf{T_2}$

Atmospheric Stability (No cloud)

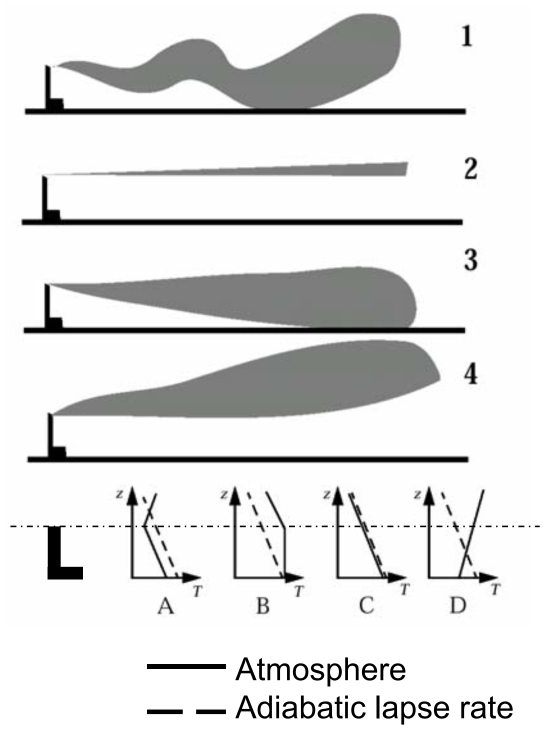
- Example 1:
- The atmospheric temperature decreases by 15 °C over 1 km in altitude. Stable or unstable?
- $dT_{atm}/dz = -15/1 = -15$ K/km $< -\Gamma$
- Decreases faster than the dry adiabatic lapse rate ($\Gamma = 9.8$ K/km) => unstable => turbulence, vertical winds

- Example 2:
- The atmospheric temperature increases by 15 °C over 1 km in altitude. Stable or unstable?
- $dT_{atm}/dz = +15/1 = 15$ K/km $> -\Gamma$
- Stable => no vertical winds
- Increased temperature with altitude: **Inversion**, extremely stable

Stable air: The atmospheric lapse rate smaller than the adiabatic lapse rate
Unstable air: The atmospheric lapse rate larger than the adiabatic lapse rate
Neutral air: $dT_{atm}/dz = -\Gamma$ is unstable

Exercise 4:1

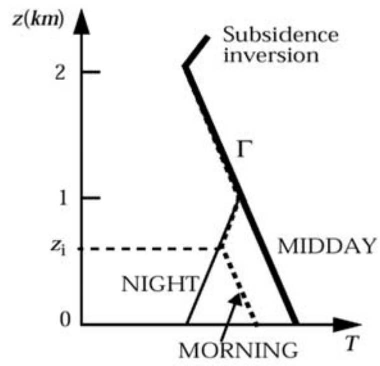
- Match each picture to the right diagram!



- Neutral: $-dT_{atm}/dz = \Gamma$; => vertical motions (like unstable air)
- **A**: Neutral at low levels, inversion at high levels. Air can move downwards – **3**
- **B**: Stable at low levels, neutral at high levels – **4**
- **C**: Neutral up- and downwards – **1**
- **D**: Inversion. Air cannot move vertically - **2**

Tropospheric Lapse Rate

- The troposphere on average stable
  - Lapse rate 6.5 K/km
  - Not the adiabatic lapse rate because
    - heat of condensation from cloud formation
    - radiation

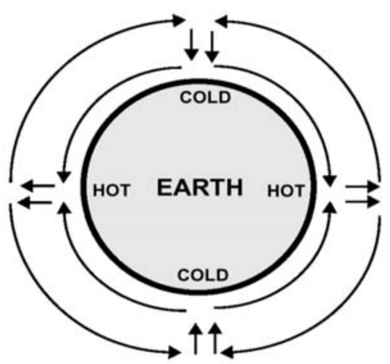


- Heating/cooling from the ground affect atmospheric stability
  - Night-time heat radiation cools the ground and the lowest atmosphere ⇒ stability
    - Often night-time inversion
  - The sun heats the ground and the lowest atmosphere ⇒ unstable
    - Vertical motion drives air parcels to the level of adiabatic equilibrium (**neutral**)

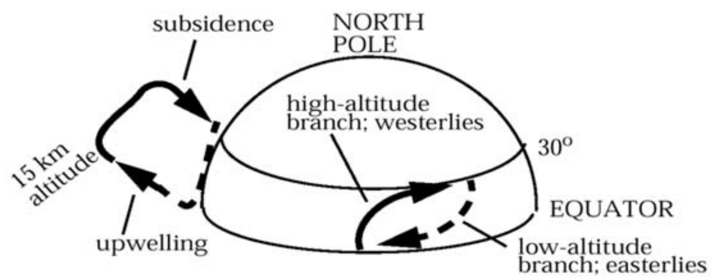
- Mixing layer
  - Continual vertical motions caused by instability
  - The height of the mixing layer varies over the day
  - The strong turbulence causes efficient transport within the mixing layer

Global Circulation

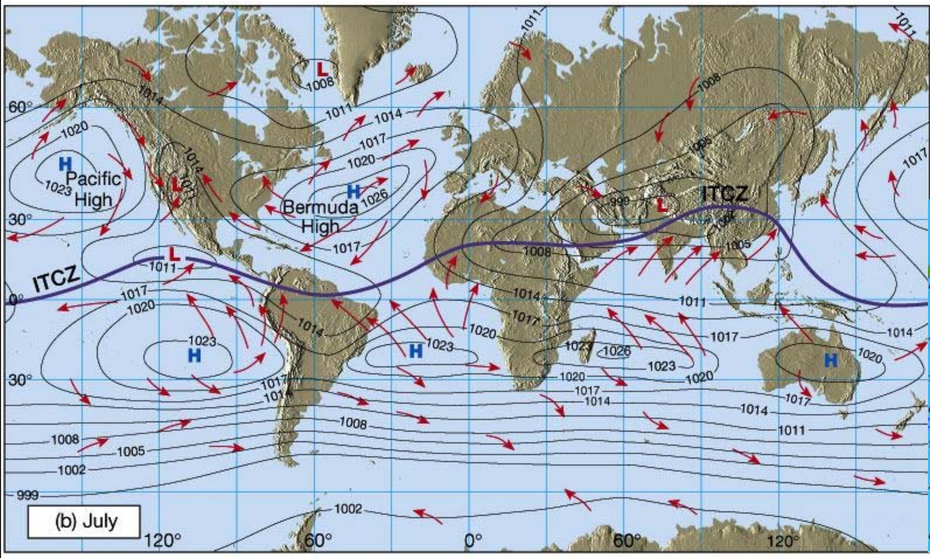
- Old model - global sea-breeze
- Coriolis force prevents equator-to-pole circulation



- The Hadley cell



Global Circulation



The Hadley cell:

- Inter-tropical convergence zone (ITCZ)
  - Strong upwind
  - Clouds and precipitation
- Subtropical high pressures (30° N,S)
  - Downwind => Dry

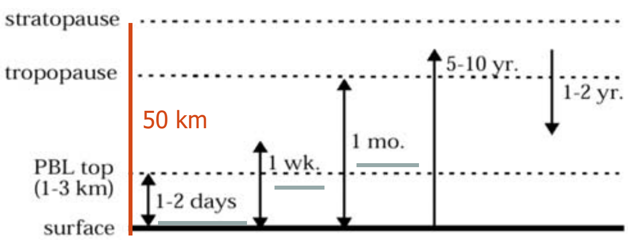
Surface winds:

- Easterly both sides of the ITCZ
- Mainly westerly at higher latitudes
  - More pronounced at SH, due to less land => less friction force => less disturbance of the geostrophic wind

Time Scales of Atmospheric Transport

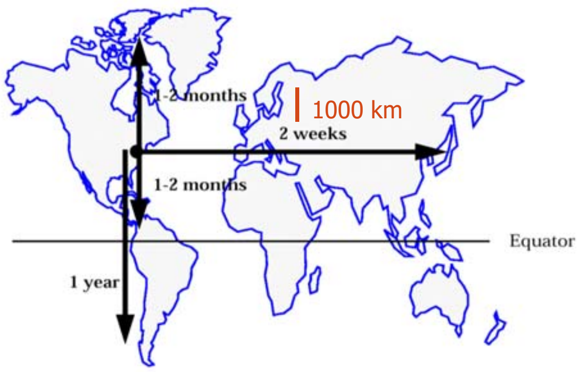
Vertically:

- Turbulent transport in the troposphere
- $\Delta t = (\Delta z)^2/(2K_z)$
  - K_z turbulent "diffusion coeff." (empirical)
  - $\langle K_z \rangle = 2 \cdot 10^5\ cm^2/s$ in troposphere
- Transport from the stratosphere faster (residence time shorter) due to $m_S << m_T$



Horizontally:

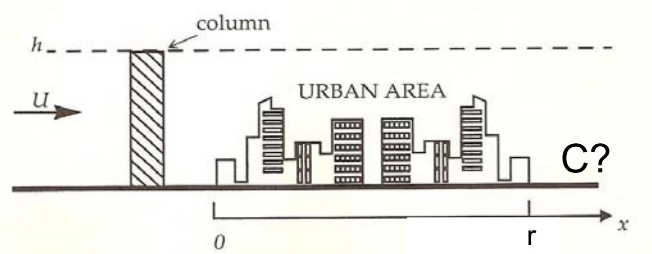
- North – south within hemisphere: 1- 2 months
- Between north – south hemispheres: 1 year
- East – west much faster than north – south
  - Caused by geostrophic flow induced by latitude-dependent temperature



Example Lagrangean Model: A village emits 3 μg SO_2 per m^2 and s. A 1.5 m/s wind blows over the village that extends 3 km in the wind direction. Air is mixed up to 1 km altitude. Calculate the SO_2 concentration added to the air by the village. (Assume P, L, D are 0)

E = 3 μg SO_2 per m^2 and s

u = 1.5 m/s; r = 3 km; h = 1 km



Mass balance eqn: $dC/dt = E/h + P - L - D = E/h$ (1)

$dC/dt = dC/dx \, dx/dt = u \, dC/dx$ (2)

Combine (1) and (2): $dC/dx = E/uh$

Integrate dC/dx: $C(r) = [r - 0]E/uh = Er/uh = 3*3000/(1.5*1000) = 6$ μg/m^3

The village increased the SO_2 concentration by 6 μg/m^3