Meteorology

‘the study of things in the air’

- Meteorological phenomena are described and quantified by variables of Earth's atmosphere:
  - temperature
  - air pressure
  - water vapor
  - mass flow

  ….and the variations and interactions of those variables, and how they change over time.

Meteorology

‘the study of things in the air’

What to learn today:

Simple models

– Obtain concentration of species

- Mass balance equation
  - Box model
  - Puff model

Literature connected with today’s lecture:

Jacob, chapter 3 - 4

Exercises:

3:1 – 3:5; 4:1 – 4:5

Winds

– Transport in the atmosphere

- Forces affecting the wind
- Flow patterns around high and low pressures
- Atmospheric stability
- General circulation → climate zones

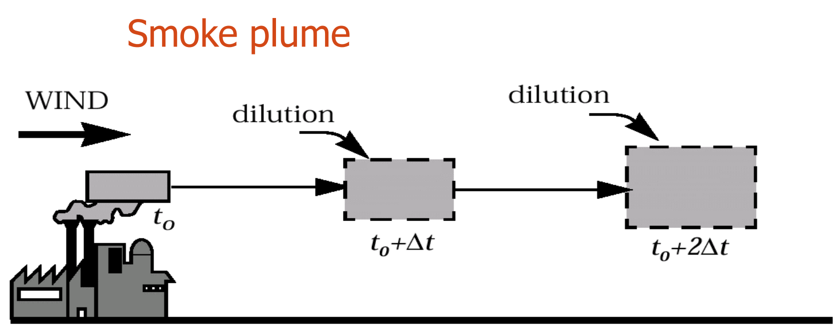
Simple Meteorological Models

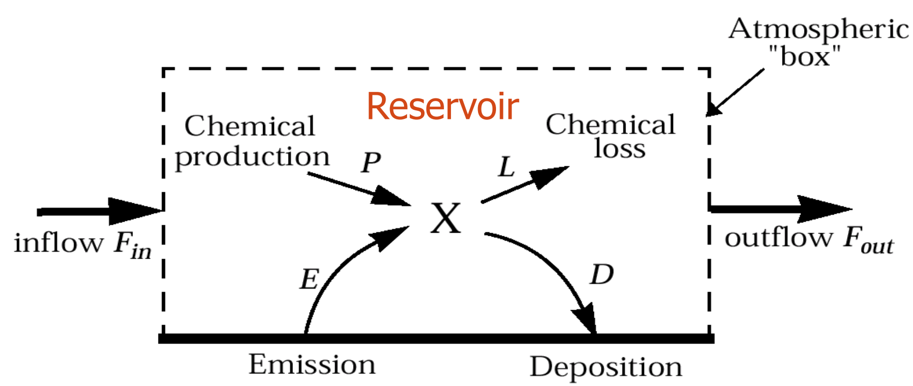
Lagranean (Puff) Model

- Follows an air parcel in the atmosphere

Eulerean (Box) Model

- Air flows in and out of the box





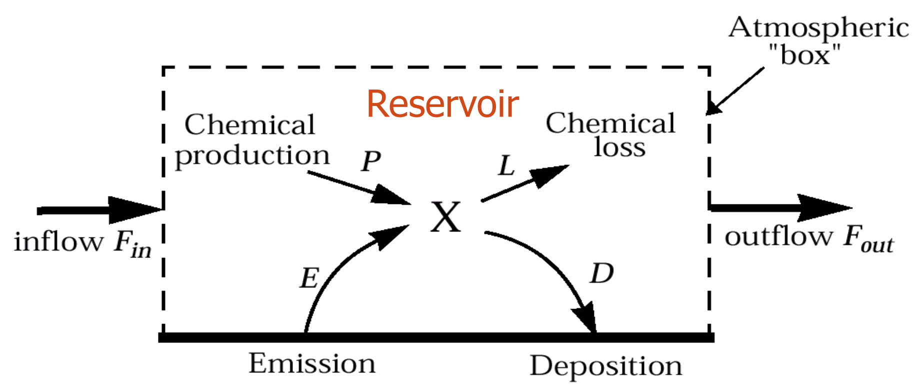
Simple Meteorological Models

Atmospheric concentration is controlled by:

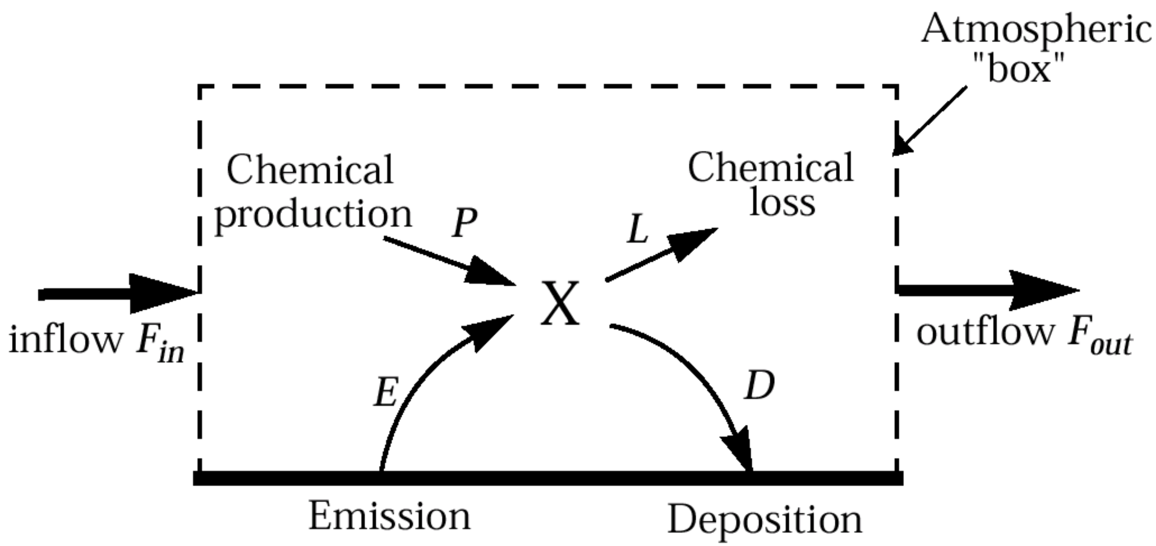
- **Emission (E):** natural and anthropogenic sources
- **Deposition (D):** dry and wet deposition
- **Transformation (P,L):** chemical reactions and phase transitions
- **Transport (F):** winds

Box model

- Imaginary box in the atmosphere
- Homogeneous concentration of X in the box is assumed in the model
  - Amount of species X (inventory)
  - Sources: E, P, F_{in}
  - Sinks: D, L, F_{out}
  - Dimension: mass/time, with X amount as mass



How can we mathematically express the mass of species X in relation to these processes?



F_{in}, F_{out}, E, D, P and L together describe the change in the mass of X

Box Model – The Mass Balance Equation

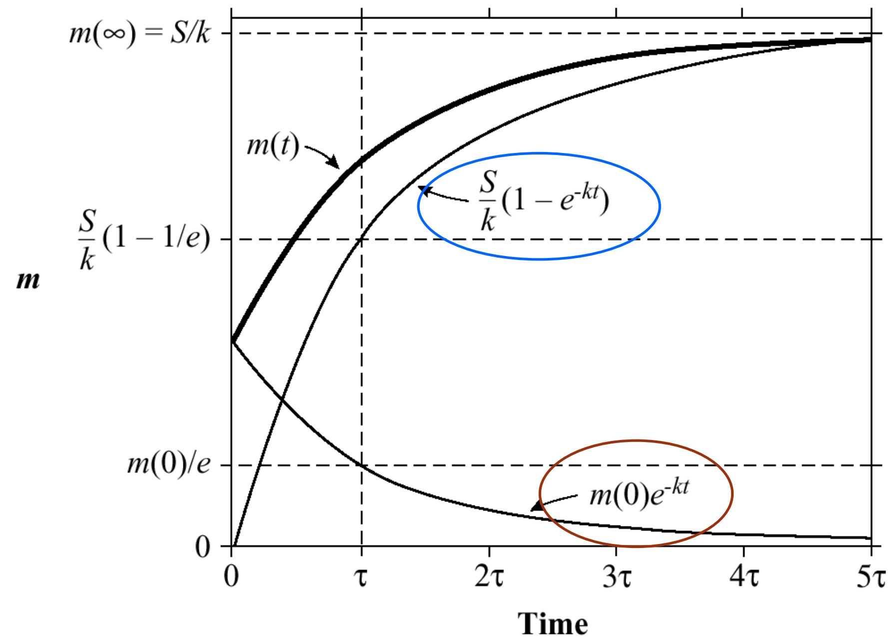
$dm/dt = \Sigma\text{sources} - \Sigma\text{sinks} = F_{in} + E + P - F_{out} - D - L$ [kg/s]

- Residence time in a box
  - Average time a species spends in the box
  - $\tau = m/(F_{out} + L + D)$ ([mass in the box]/[sinks])

- First order sinks (proportional to m) 'the more one has, the more one can lose'
  - Sinks often proportional to amount in box ($\Sigma\text{sinks} = km$)
  - $F_{out} + L + D = (k_{out} + k_l + k_d)m = km$
  - Loss rate constant: $k = (F_{out} + L + D)/m = 1/\tau$

Box Model – Common Special Case

- Sources independent of m
- Sinks proportional to m



Mass balance equation:

$dm/dt = \Sigma sources - \Sigma sinks =$

$= F_{in} + E + P - F_{out} - D - L$

Sources $F_{in} + E + P = S$ (S indep. of m)

Sinks $F_{out} + D + L = km$ (sinks prop. m)

$dm/dt = S - km$

Compute m(t)!

$\int dm/(S-km) = \int dt \Rightarrow$

$m(t) = m(0)e^{-kt} + S/k(1-e^{-kt})$

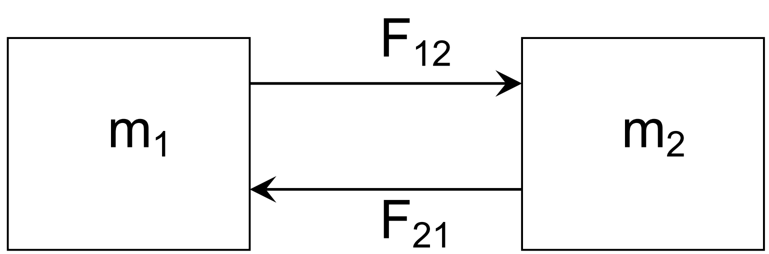
Decay of initial condition   Approach to steady state

$t \rightarrow \infty \Rightarrow m \rightarrow S/k$

Multiple Box Models

- A single box model sometimes oversimplifies a problem
- Multi-box models allow concentration to vary between boxes

Two-box Model



- Mass balance equation for box 1:

$$\frac{dm_1}{dt} = E_1 + P_1 - L_1 - D_1 - F_{12} + F_{21}$$

- For first order process:
  - $F_{12} = k_{12}m_1$
  - $F_{21} = k_{21}m_2$
- Coupled differential equations:

$$\frac{dm_1}{dt} = E_1 + P_1 - L_1 - D_1 - k_{12}m_1 + k_{21}m_2$$

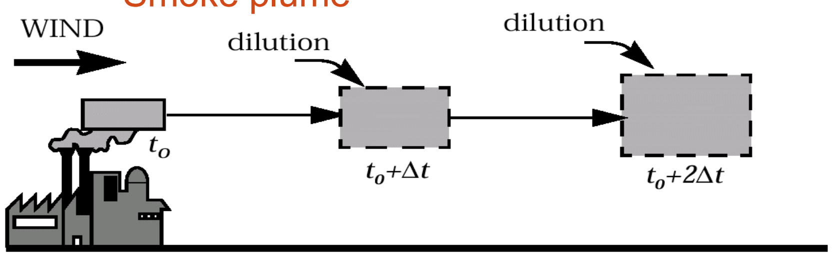
$$\frac{dm_2}{dt} = E_2 + P_2 - L_2 - D_2 + k_{12}m_1 - k_{21}m_2$$

Can be extended to 3 (or more) boxes

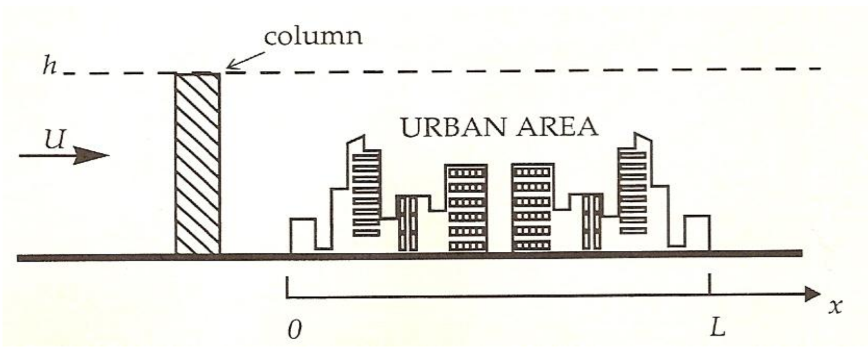
Puff Model

- **Box model:** Air flows in and out of the box (Eulerean model)
- **Puff models:** Follows an air parcel in the atmosphere (Lagrangean model)

Smoke plume



- Mass bal. eqn. Puff model:
  - d[X]/dt = E + P - D - L
  - Advantage: $F_{in} = F_{out} = 0$
  - Disadvantage: Limited range due to turbulence that diffuses the air parcel
- Common applications:
  - Smoke plumes
    - $d[X]/dt = E+P-D-L-k_{dil}([X]-[X]_b)$
    - k_{dil} = dilution constant
  - Column model
    - d[X]/dt = E/h+P-D-L
    - E = emission per area and time (kg m^{-2} s^{-1})
    - h = height of the column (mixing height)



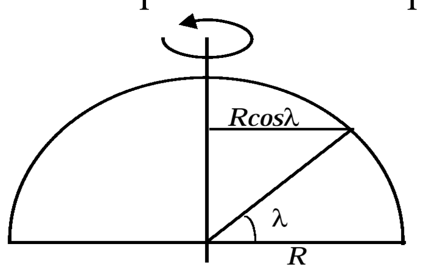
Transport in the Atmosphere

Forces that affect winds:

- Gradient force - Horizontal pressure gradient, strong winds (high-, low pressure)
- Gravitational force - induces vertical air motions related to density
- Coriolis force – Caused by the rotation of the earth
- Friction force – Acts on winds in contact with the ground

Coriolis Force

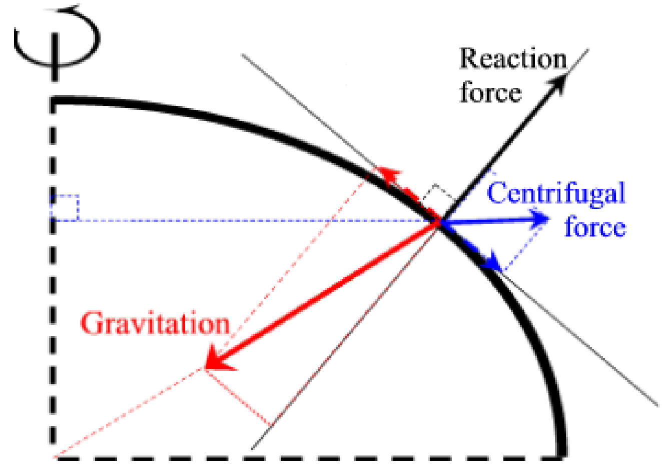
- Earth is spinning around its axis
  - Points near the equator travels faster than points near the poles



$v_E = 2\pi R\cos\lambda/t$

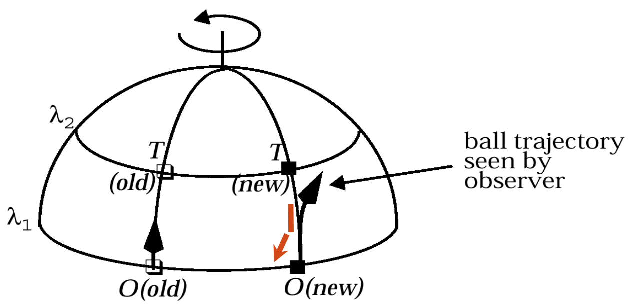
v_E(Lund) ≈ 260 m/s
v_E(Equator) ≈ 460 m/s

- Coriolis force along latitude



Winds are also deflected by the centrifugal force

- Coriolis force along longitude



Motion west – east in NH

⇒ increased velocity

⇒ increased centrifugal force

⇒ Bends towards equator (right)

East - west

bends towards the north pole (right)

Winds along both longitude and latitude bend

SH: Left NH: Right

Coriolis Force

- Coriolis acceleration

$$\gamma_c = 2\omega v \sin\lambda$$

- Resulting displacement

$$\Delta Y = \omega(\Delta X)^2 \sin\lambda / v$$

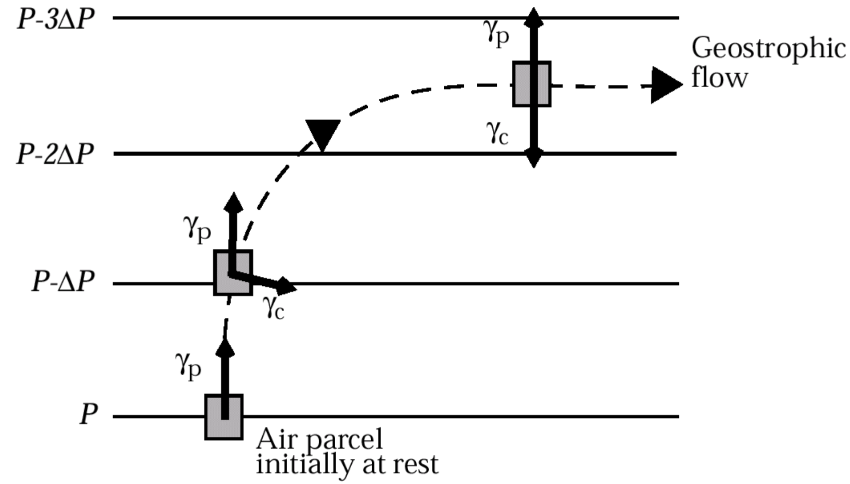
ω = $2\pi/T$ (angular velocity)
T = 24 hours
v = velocity relative the earth
ΔX = distance
λ = latitude

Example (λ = 42°):

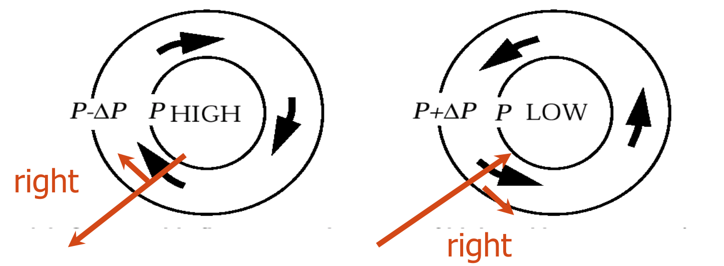
1. Snow ball v = 20 km/h ΔX = 10 m: ΔY = 1mm
2. Missile v = 2000 km/h ΔX = 1000 km: ΔY = 100km

Geostrophic Wind

- Pressure gradient induces a wind
  - Motion induces Coriolis force
  - Air trajectory bends until balance between gradient and Coriolis forces
  - Wind along isobar
  - No air transport to/from low/high pressure!

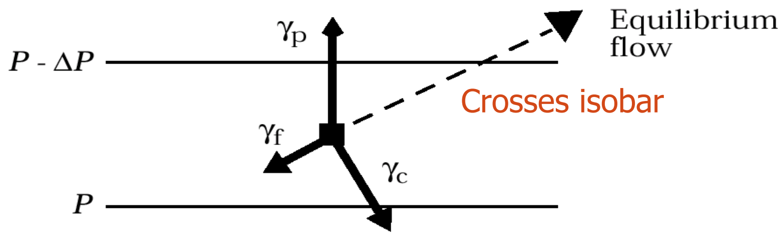


- Geostrophic wind around high and low pressures (northern hemisphere)



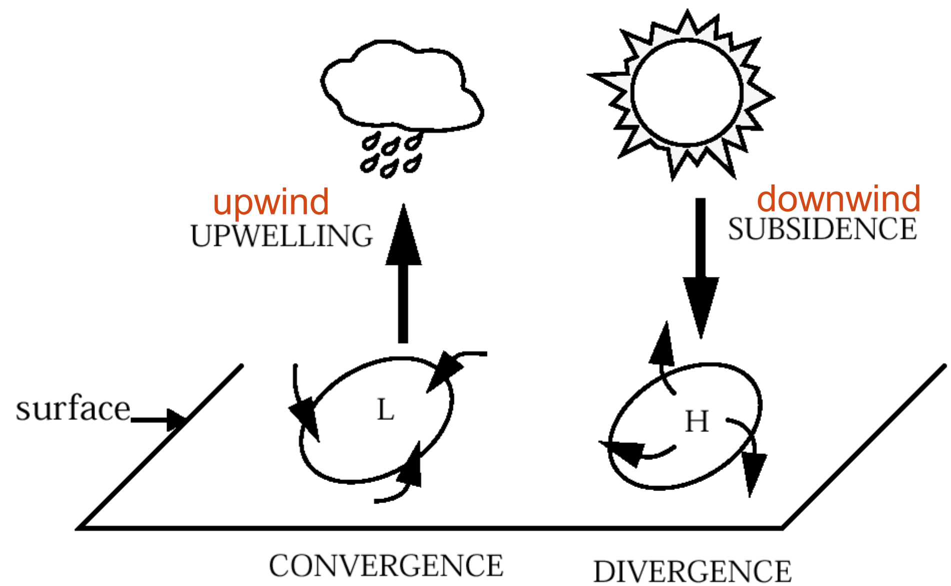
Friction Force

- Effect of friction against the ground



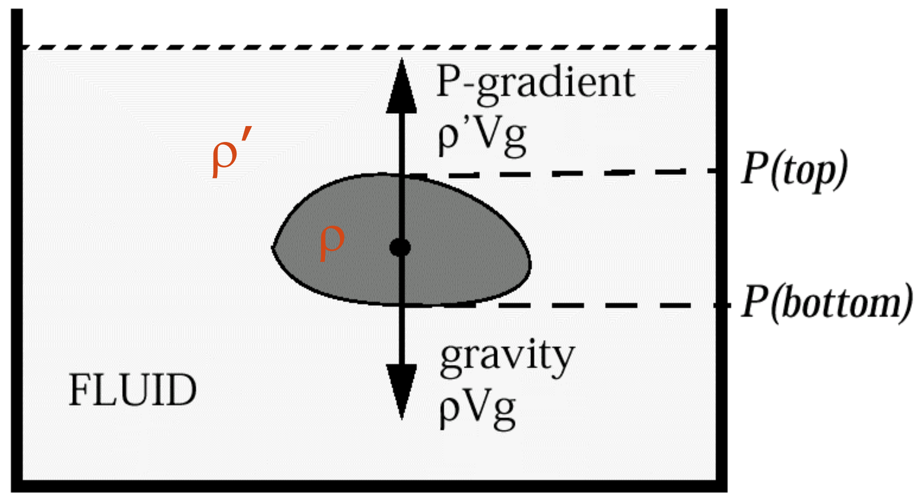
- Friction acts against the motion and reduces the speed
  - reduced Coriolis force
  - close to the ground, winds cross the isobars

- High and low pressures
  - Wind to low pressures
    - Convergence - Upwind in low pressures
    - The air ascends, expands, and cool – Clouds form
  - Wind from high pressure close to the ground
    - Divergence - Subsidence in high pressures
    - Air descends, compresses, and warmed - Clear weather



Vertical Transport

- **A fluid at equilibrium:** The force from a pressure gradient acting on a volume element is balanced by the gravitational force
  - Gradient force: $F_g = \rho' V g$
- Convection (buoyancy)
  - Caused by difference in density :
  - Gravitational force: $mg = -\rho V g$
- Resulting force: $F = (\rho' - \rho) V g$



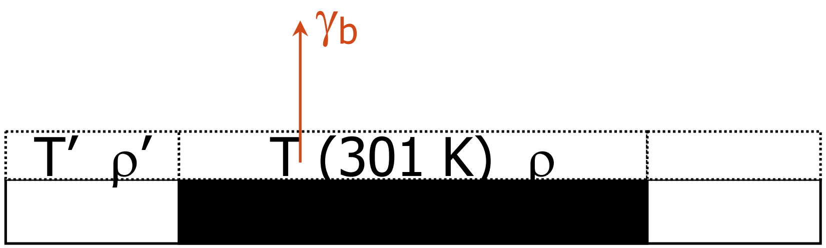
Buoyant acceleration:

$$\gamma_b = F/m = (\rho' - \rho) g / \rho$$

Exercise 4-1 (Jacob)

Assume that the air surface temperature over a black parking lot is 1 K higher than in the surrounding air (300 K).

Calculate the buoyant acceleration!



Bouyant acceleration: $\gamma_b = (\rho' - \rho) g / \rho$

From the ideal gas law: $\rho = PM_a / RT$

$\gamma_b = (T - T') g / T'$

$\gamma_b = 0.033 \text{ m/s}^2$

Calculate the upward wind speed after 1 s?

$w = t\,\gamma_b = 3.3 \text{ cm/s}$

Comparison:

Global circulation: $w \approx 0.1$ cm/s

Cumulus clouds: w several m/s

A small increase in temperature makes a huge impact on vertical mixing!

Does the velocity continue to increase? →

Atmospheric stability

Adiabatic Lapse Rate

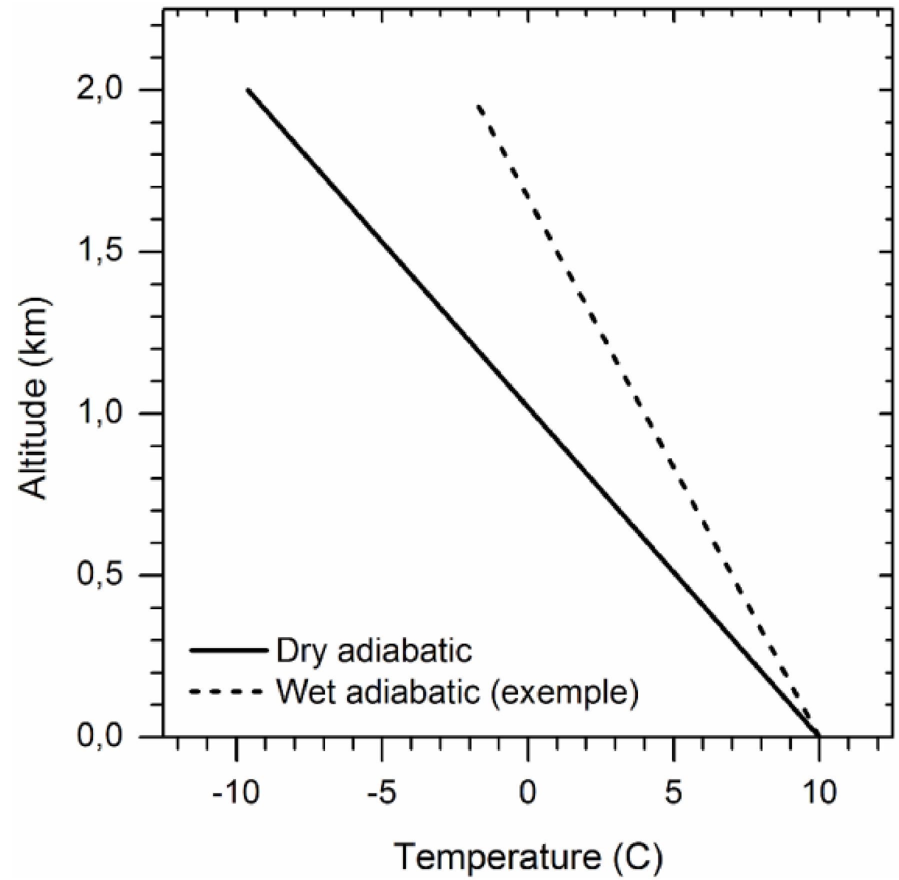
- Small heat exchange in vertical transport ($dQ \approx 0$) $\Rightarrow$ Adiabatic process
  - Upwards $\Rightarrow$ expansion $\Rightarrow$ cooling
  - Downwards $\Rightarrow$ compression $\Rightarrow$ warming

- Dry adiabatic lapse rate

  Can be shown that:

  $\Gamma = -dT/dz = g/C_p = 9.8$ K/km

- Wet adiabatic lapse rate
  - Cloud formation
  - heat of condensation released
  - weaker gradient; Γ_w 2-7 K/km

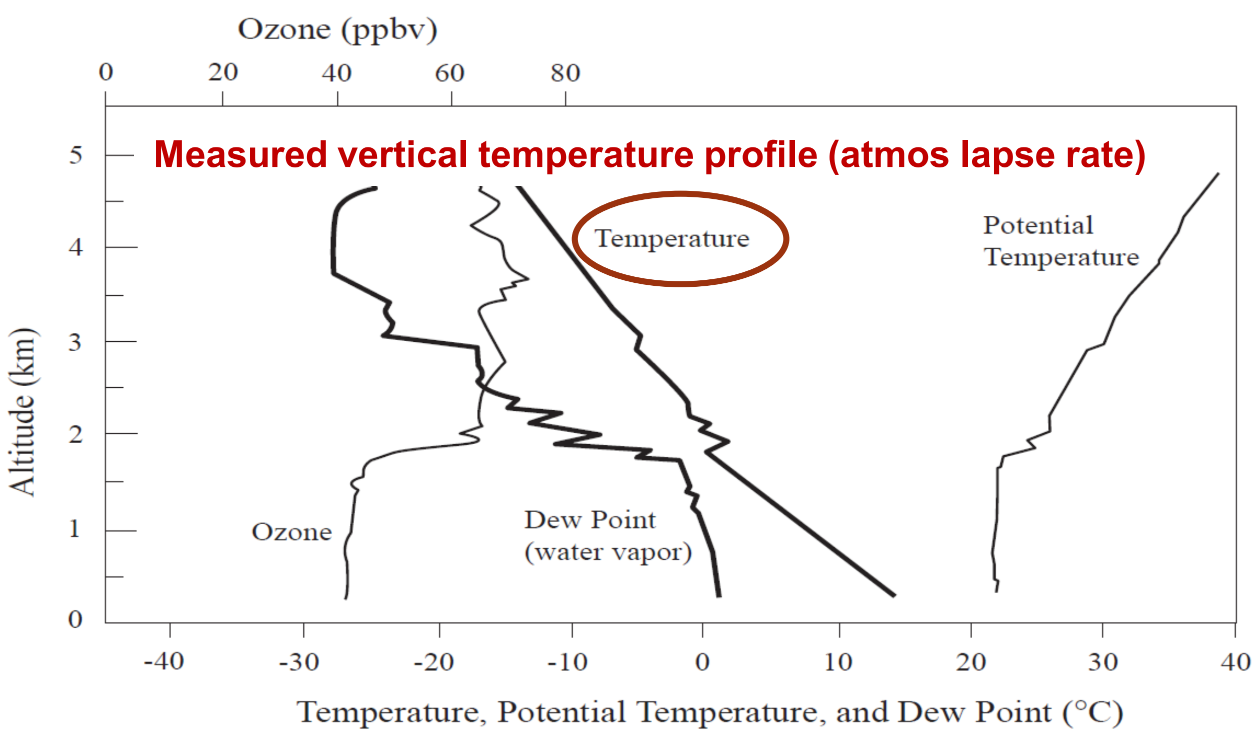


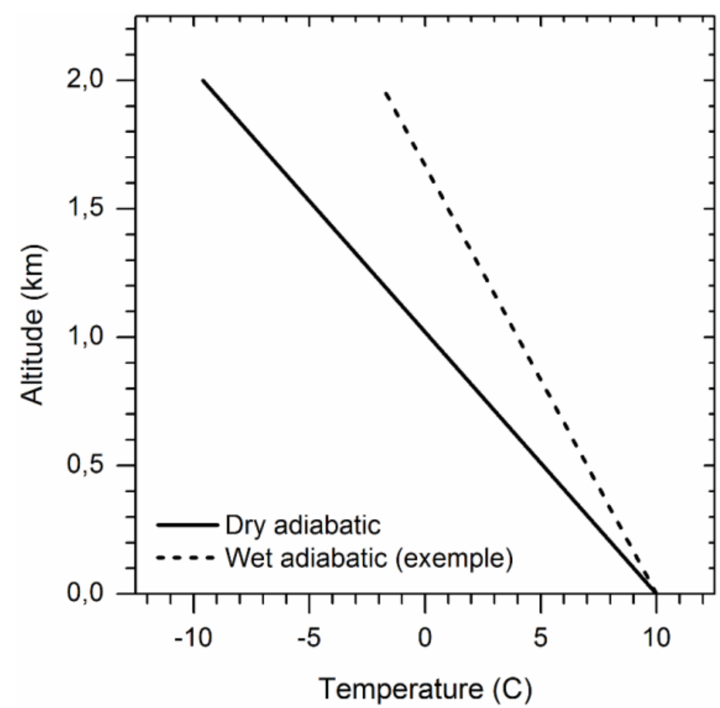
Air masses in vertical motion follow the adiabatic lapse rate (dry or wet)

Atmospheric Stability

Sometimes strong vertical motions and turbulence – sometimes not – WHY?

It depends on the atmospheric lapse rate!





Atmospheric Stability

Sometimes strong vertical motions and turbulence – sometimes not – WHY?

Disturbance:

An air mass at height z_a is lifted to z_b

⇒ Expands and cools adiabatically to **T**

At z_a (initially): $\mathbf{T_a}$

At z_b:

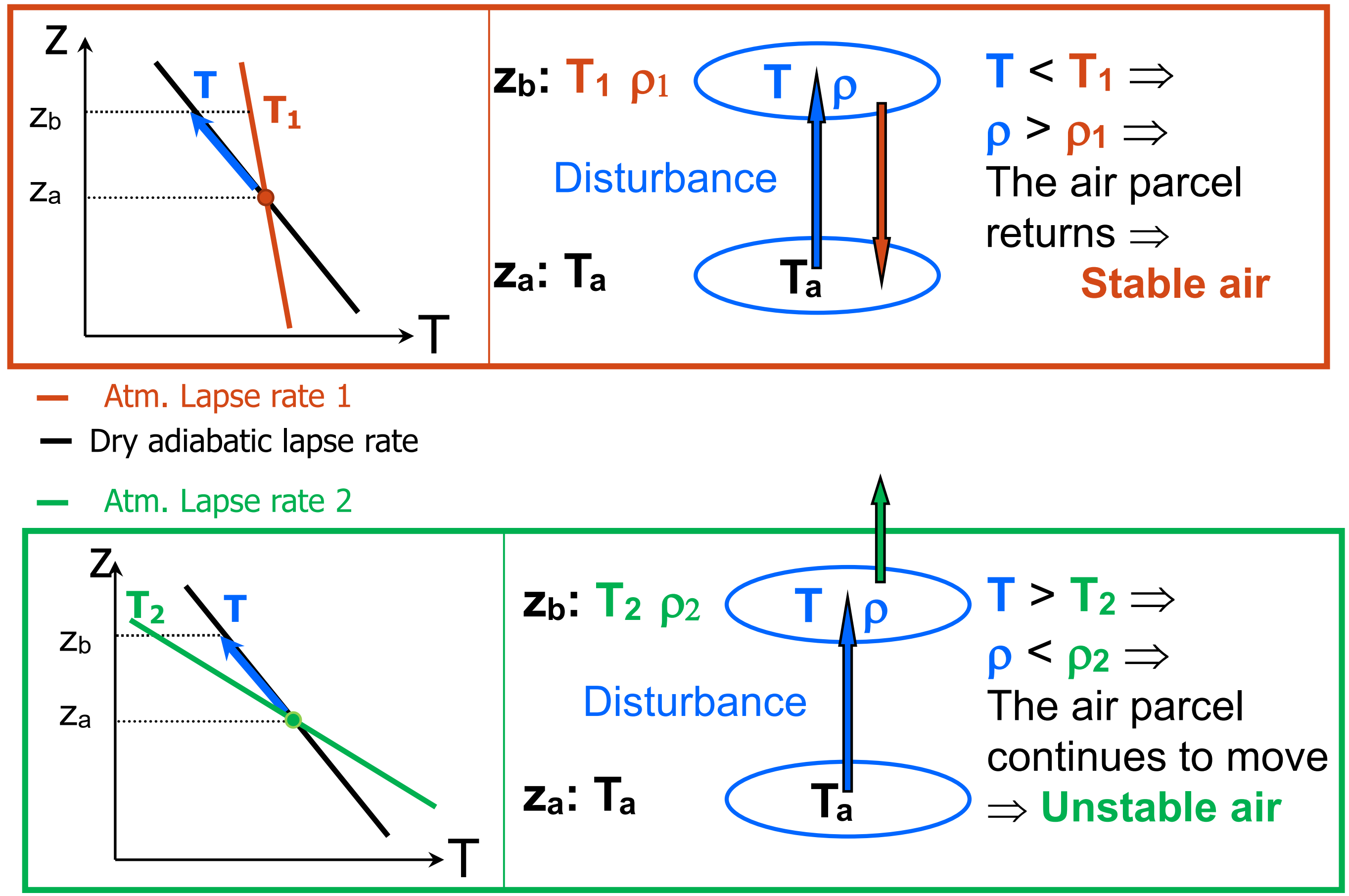
Disturbed air mass:

$\mathbf{T} = T_a - \Gamma(z_b - z_a)$

Surrounding atmosphere:

Case 1: $\mathbf{T_1}$

Case 2: $\mathbf{T_2}$



Atmospheric Stability (No cloud)

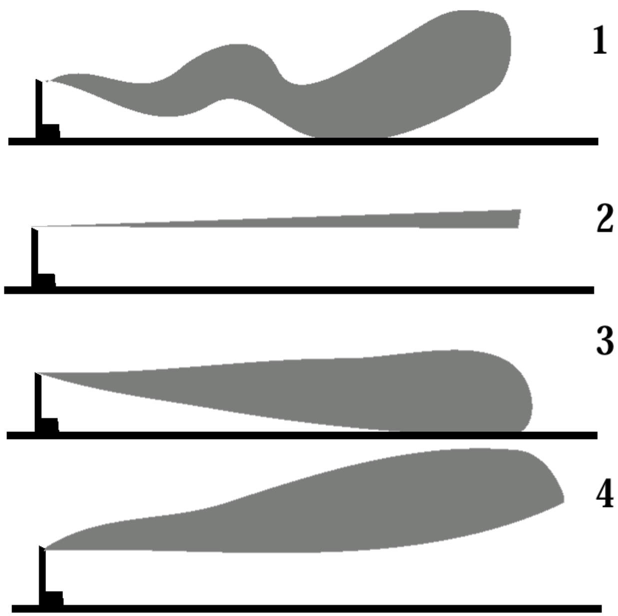
- Example 1:
- The atmospheric temperature decr by 15 °C over 1 km in altitude. Stable or unstable?
- $dT_{atm}/dz = -15/1 = -15$ K/km $< -\Gamma$
- Decreases faster than the dry adiabatic lapse rate ($\Gamma = 9.8$ K/km) => unstable => turbulence, vertical winds

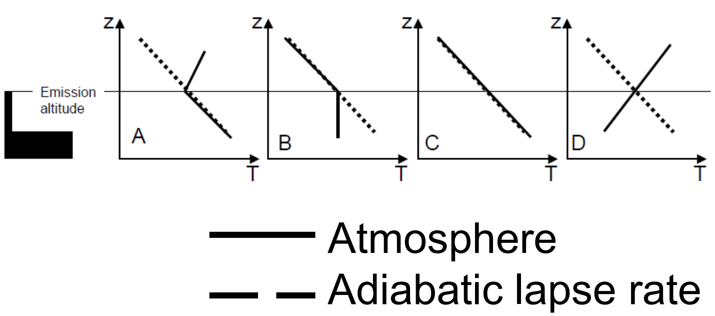
- Example 2:
- The atmospheric temperature increases by 15 °C over 1 km in altitude. Stable or unstable?
- $dT_{atm}/dz = +15/1 = 15$ K/km $> -\Gamma$
- Stable => no vertical winds
- Increased temperature with altitude: **Inversion**, extremely stable

Stable air: The atmospheric lapse rate smaller than the adiabatic lapse rate
Unstable air: The atmospheric lapse rate larger than the adiabatic lapse rate
Neutral air: $dT_{atm}/dz = -\Gamma$ is unstable

Exercise 4:1

- Match each picture to the right diagram!

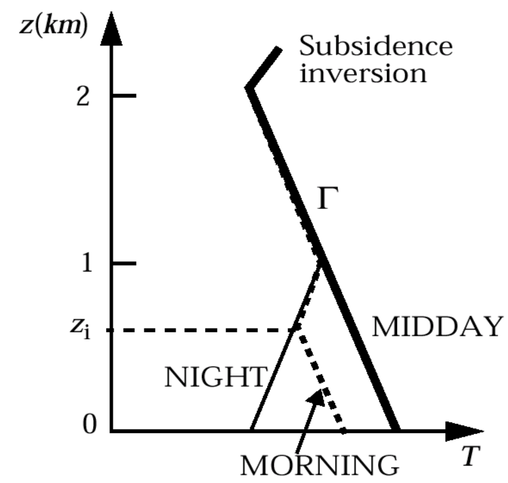




- Neutral: - $dT_{atm}/dz = \Gamma$; => vertical motions (like unstable air)
- **A**: Neutral at low levels, inversion at high levels. Air can move downwards – **3**
- **B**: Stable at low levels, neutral at high levels – **4**
- **C**: Neutral up- and downwards – **1**
- **D**: Inversion. Air cannot move vertically - **2**

Tropospheric Lapse Rate

- The troposphere on average stable
  - Lapse rate 6.5 K/km
  - Lower than the adiabatic lapse rate since
    - heat of condensation from cloud formation
    - radiation

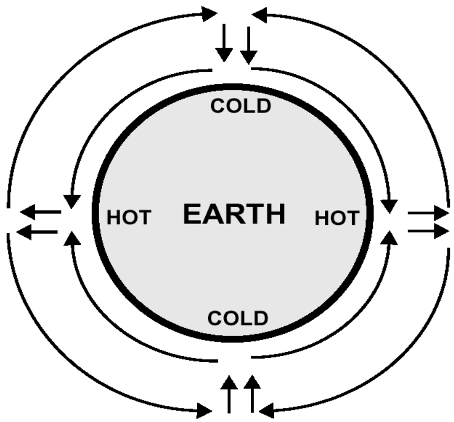


- Heating/cooling from the ground affect atmospheric stability
  - Night-time heat radiation cools the ground and the lowest atmosphere ⇒ stability
    - Often night-time inversion
  - The sun heats the ground and the lowest atmosphere ⇒ unstable
    - Vertical motion drives air parcels to the level of adiabatic equilibrium (**neutral**)

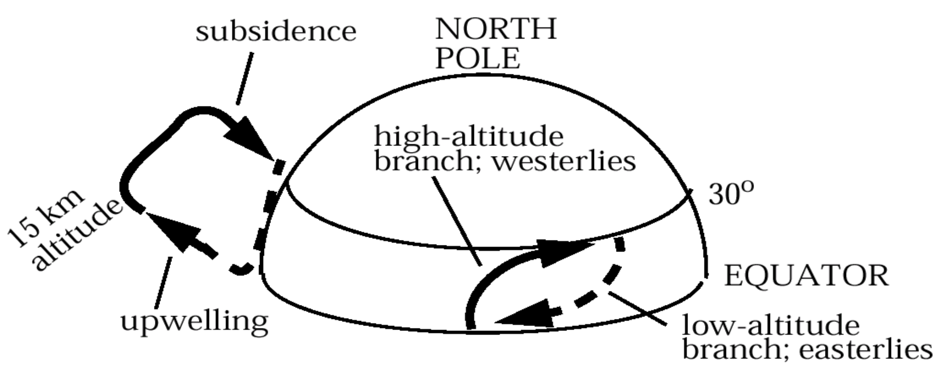
- Lowest 1-2 km – a mixing layer
  - Continual vertical motions caused by instability
  - The height of the mixing layer varies over the day
  - The strong turbulence causes efficient transport within the mixing layer
  - Rather homogeneous composition throughout this layer

Global Circulation

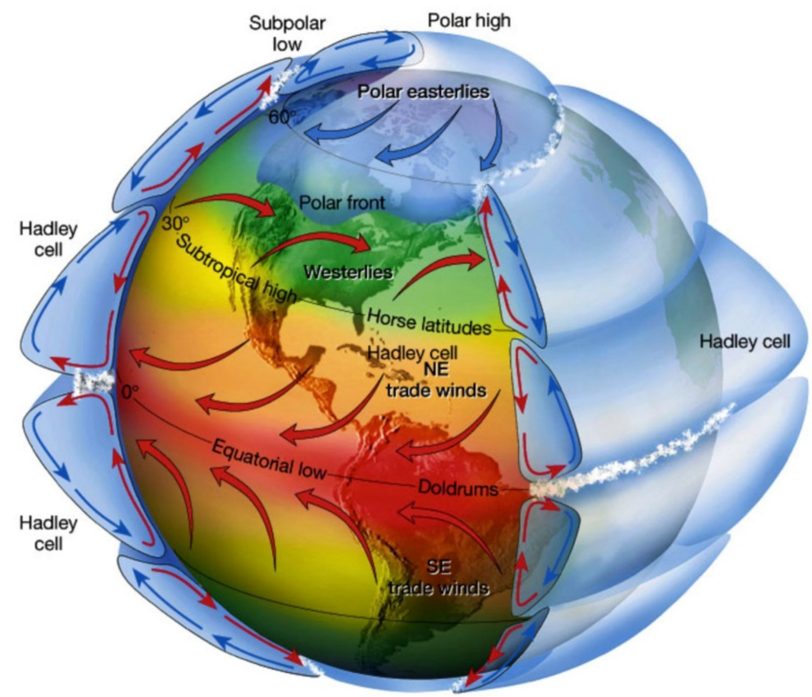
- Old model - global sea-breeze



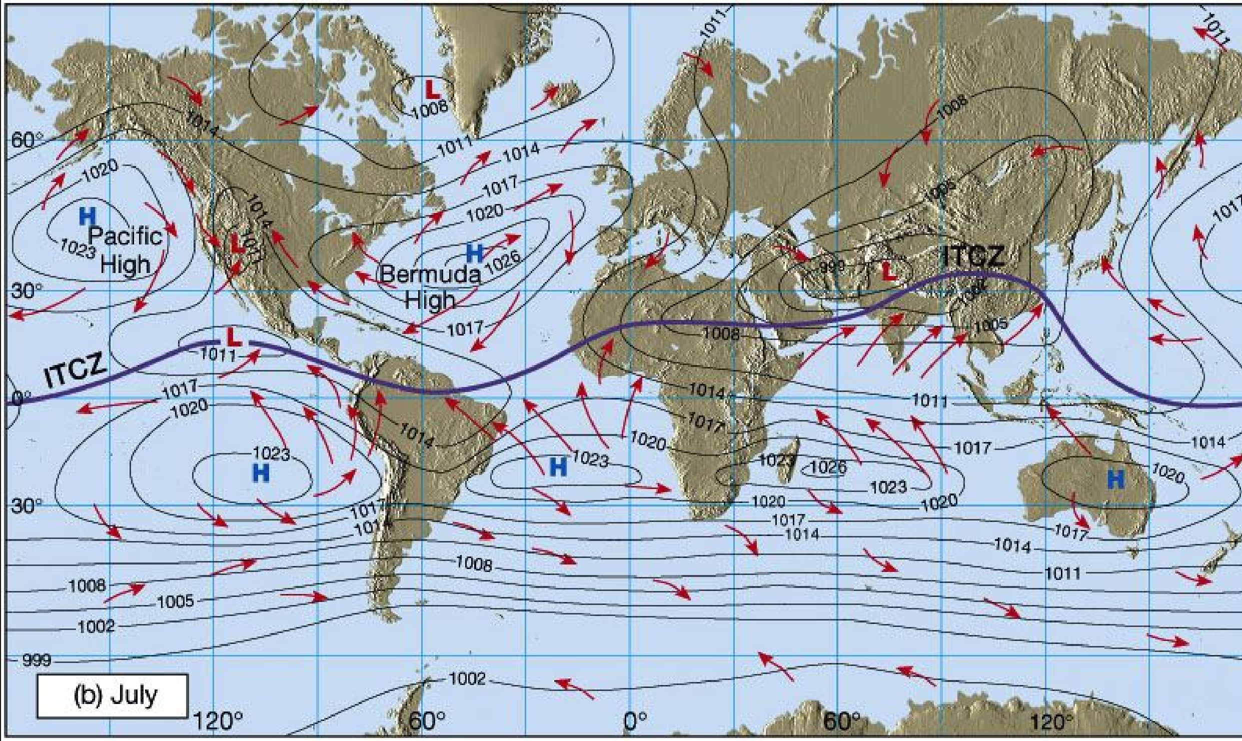
- Coriolis force prevents equator-to-pole circulation

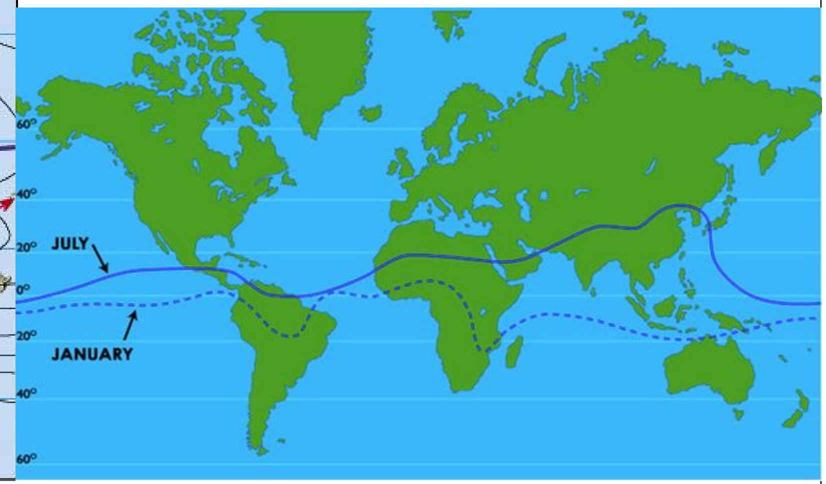


- 3 large circulation cells per hemisphere
- Separation into climate zones



Global Circulation





The Hadley cell:

- Inter-tropical convergence zone (ITCZ)
  - Strong upwind
  - Clouds and precipitation
- Subtropical high pressures (30° N,S)
  - Downwind => Dry

Surface winds:

- Easterly both sides of the ITCZ
- Mainly westerly at higher latitudes
  - More pronounced at SH, due to less land => less friction force => less disturbance of the geostrophic wind

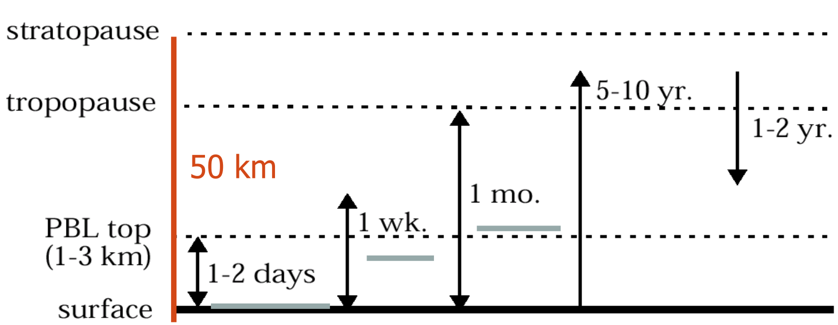
Time Scales of Atmospheric Transport

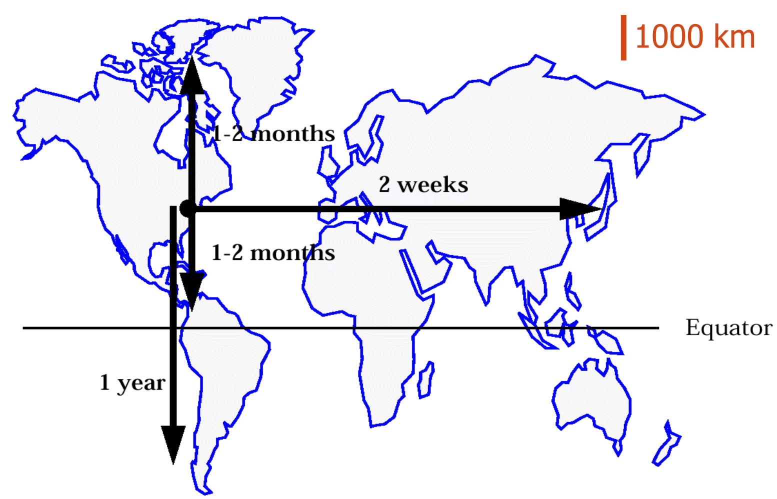
Vertically:

- Turbulent transport in the troposphere
- $\Delta t = (\Delta z)^2/(2K_z)$
  - K_z turbulent "diffusion coeff." (empirical)
  - $\langle K_z \rangle = 2 \cdot 10^5\ cm^2/s$ in troposphere
- Transport from the stratosphere faster (residence time shorter) due to $m_S << m_T$

Horizontally:

- North – south within hemisphere: 1- 2 months
- Between north – south hemispheres: 1 year
- East – west much faster than north – south
  - Caused by geostrophic flow induced by latitude-dependent temperature





Exercise 3-3 in Jacob: The sink of CFC-12 (CF_2Cl_2) is exclusively photolysis (residence time 100 years). Year 1980 the concentration was 400 pptv and the rate of increase 4% per year. Calculate the 1980 CFC-12 emission!

Use the mass balance equation:

$$\frac{dm}{dt} = F_{in} + E + P - F_{ut} - L - D$$

Box: Entire atmosphere $\Rightarrow F_{in} = F_{out} = 0$
$L + D = L = km$ (sink: photolysis only)
$E + P = E$ (no chemical production)

$$\frac{dm}{dt} = E - km$$

dm/dt is given by a relative measure: $k_{inc} = 4\%$ per year):

$$\frac{dm}{dt} = k_{inc} \times m$$

$k = 1/\tau = 0.01\ \text{year}^{-1}$

Sink (photolysis):

Emissions (from mass balance equation): $\boxed{E = \frac{dm}{dt} + km = (k_{inc} + k)m}$

Enter numbers: $E = (0.04\ \text{year}^{-1} + 0.01\ \text{year}^{-1})\ m$

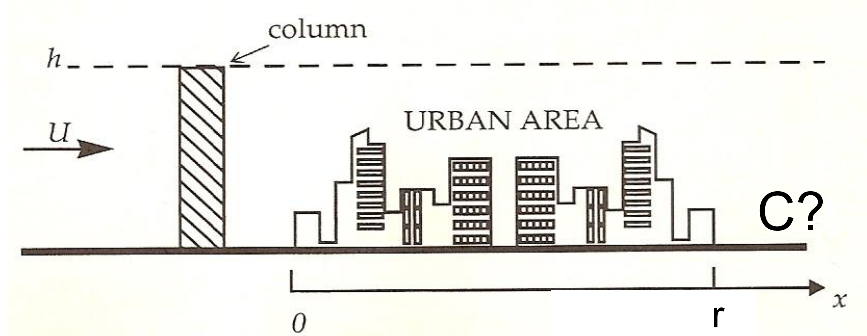
$$m = M_{CFC} n_{CFC} = \left[C_{CFC} = \frac{n_{CFC}}{n_a} \right] = M_{CFC} C_{CFC} n_a = M_{CFC} C_{CFC} \frac{m_a}{M_a} = M_{CFC} C_{CFC} \frac{4\pi R^2 P}{M_a g}$$

Result: $E = 4.4 \times 10^8$ kg/year

Example Lagrangean Model: A village emits 3 μg SO_2 per m^2 and s. A 1.5 m/s wind blows over the village that extends 3 km in the wind direction. Air is mixed up to 1 km altitude. Calculate the SO_2 concentration added to the air by the village. (Assume P, L, D are 0)

$E = 3\ \mu g\ SO_2$ per m^2 and s

$u = 1.5$ m/s; $r = 3$ km; $h = 1$ km



Mass balance eqn: $dC/dt = E/h + P - L - D = E/h$ (1)

$dC/dt = dC/dx\ dx/dt = u\ dC/dx$ (2)

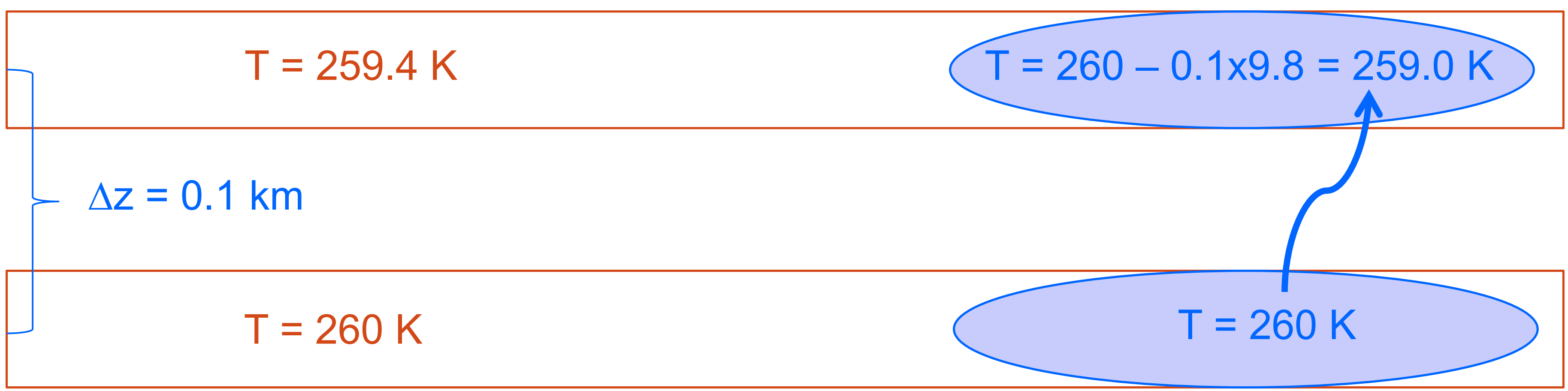
Combine (1) and (2): $dC/dx = E/uh$

Integrate dC/dx: $\underbrace{C(r) - C(0)}_{\text{Added concentration}} = [r - 0]E/uh = Er/uh = 3{*}3000/(1.5{*}1000) = 6\ \mu g/m^3$

The village increased the SO_2 concentration by $6\ \mu g/m^3$

A dry air mass with the same temperature (260 K) as the surrounding air is lifted 0.1 km. The surrounding air temperature at the new level is 259.4 K.

Will this air mass continue to rise or fall back?



Hint: Air density is inversely proportional to the temperature